Introduction

Physics is one of the most fundamental of the sciences. Scientists of all disciplines use the ideas of physics, including chemists who study the structure of molecules, paleontologists who try to reconstruct how dinosaurs walked, and climatologists who study how human activities affect the atmosphere and oceans. Physics is also the foundation of all engineering and technology.

No engineer could design a flat-screen TV, an interplanetary spacecraft, or even a better mousetrap without first understanding the basic laws of physics. The study of physics is also an adventure. You will find it challenging, sometimes frustrating, occasionally painful, and often richly rewarding. If you've ever wondered why the sky is blue, how radio waves can travel through empty space, or how a satellite stays in orbit, you can find the answers by using fundamental physics. You will come to see physics as a towering achievement of the human intellect in its quest to understand our world and ourselves.

Physics is an experimental science. Experiments require measurements, and we generally use numbers to describe the results of measurements. Any number that is used to describe a physical phenomenon quantitatively is called a **physical quantity**. For example, two physical quantities that describe you are your weight and your height. Some physical quantities are so fundamental that we can define them only by describing how to measure them. Such a definition is called an operational definition. Two examples are measuring a distance by using a ruler and measuring a time interval by using a stopwatch. In other cases we define a physical quantity by describing how to calculate it from other quantities that we can measure. Thus we might define the average speed of a moving object as the distance travelled (measured with a ruler) divided by the time of travel (measured with a stopwatch).

International System (SI)

To make accurate, reliable measurements, we need units of measurement that do not change and that can be duplicated by observers in various locations. The system of units used by scientists and engineers around the world is commonly called "the metric system," but since 1960 it has been known officially as the **International System**, or **SI** (the abbreviation for its French name, Système International).

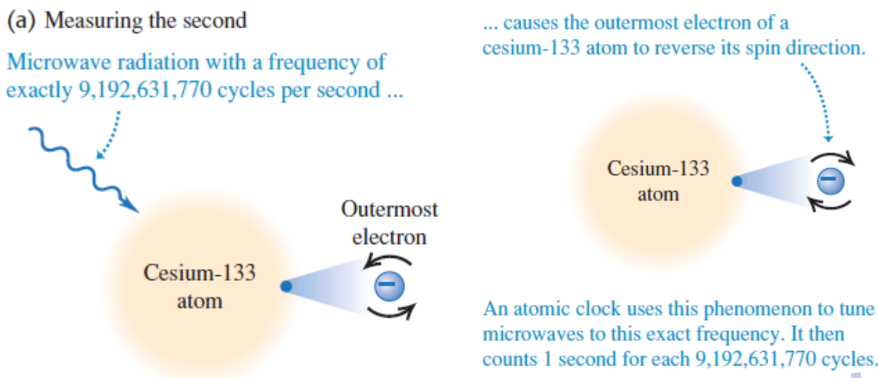
Appendix A gives a list of all SI units as well as definitions of the most fundamental units. University Physics with modern physics. -- 13th ed. by Sears and Mark W. Zemansky, Hugh D. Young, Roger A. Freedman

TABLE 1.4 Prefixes for SI Units

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deka	da
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

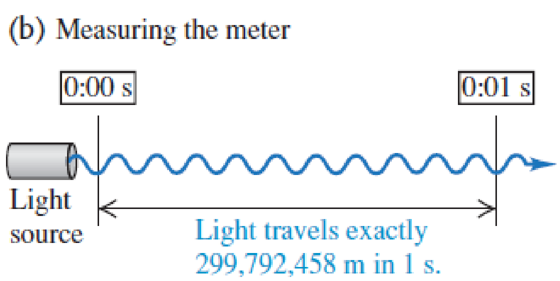
Time

From 1889 until 1967, the unit of time was defined as a certain fraction of the mean solar day, the average time between successive arrivals of the sun at its highest point in the sky. The present standard, adopted in 1967, is much more precise. It is based on an atomic clock, which uses the energy difference between the two lowest energy states of the cesium atom. When bombarded by microwaves of precisely the proper frequency, cesium atoms undergo a transition from one of these states to the other. One second (abbreviated ***s***) is defined as the time required for 9,192,631,770 cycles of this microwave radiation (Fig. 1.3a).

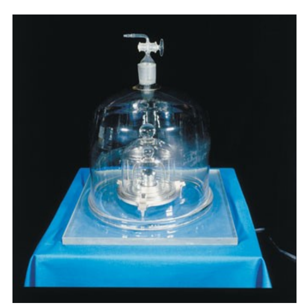


Length

In 1960 an atomic standard for the meter was also established, using the wavelength of the orange-red light emitted by atoms of krypton in a glow discharge tube. Using this length standard, the speed of light in vacuum was measured to be 299,792,458 m s. In November 1983, the length standard was changed again so that the speed of light in vacuum was defined to be precisely 299,792,458 m s. Hence the new definition of the meter (abbreviated ***m***) is the distance that light travels in vacuum in 1/ 299,792,458 second (Fig. 1.3b). This provides a much more precise standard of length than the one based on a wavelength of light.



Mass



The standard of mass, the kilogram (abbreviated ***kg***), is defined to be the mass of a particular cylinder of platinum–iridium alloy kept at the International Bureau of Weights and Measures at Sèvres, near Paris (Fig. 1.4). An atomic standard of mass would be more fundamental, but at present we cannot measure masses on an atomic scale with as much accuracy as on a macroscopic scale. The gram (which is not a fundamental unit) is 0.001 kilogram.

The British System

Finally, we mention the British system of units. These units are used only in the United States and a few other countries, and in most of these they are being replaced by SI units. British units are now officially defined in terms of SI units, as follows:

Length: **1 inch** = 2.54 cm (exactly)

Force: **1 pound** = 4.448221615260 newtons (exactly)

Unit Consistency and Conversions

We use equations to express relationships among physical quantities, represented by algebraic symbols. An equation must always be dimensionally consistent. You can't add apples and automobiles; two terms may be added or equated only if they have the same units.

For example, if a body moving with constant speed travels a distance ***d*** in a time ***t*,** these quantities are related by the equation, ***d*** = ***v t***

For example, ***d*** might represent a distance of 10 m, ***t*** a time of 5 s, and a ***v*** speed of 2m/s

If ***d*** is measured in meters, then the product must also be expressed in meters.

Using the above numbers as an example, we may write 10 m = (2m/s) (5 s)

***Examples*;**

Example 1.1 Converting speed units

The world land speed record is 763.0 mi/h, set on October 15, 1997, by Andy Green in the jet-engine car *Thrust SSC*. Express this speed in meters per second.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: We need to convert the units of a speed from mi/h to m/s. We must therefore find unit multipliers that relate (i) miles to meters and (ii) hours to seconds. In Appendix E (or inside the front cover of this book) we find the equalities 1 mi = 1.609 km, 1 km = 1000 m, and 1 h = 3600 s. We set up the conversion as follows, which ensures that all the desired cancellations by division take place:

$$763.0\text{ mi/h} = \left(763.0\,\frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{1.609\,\cancel{\text{km}}}{1\,\cancel{\text{mi}}}\right)\left(\frac{1000\text{ m}}{1\,\cancel{\text{km}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\text{ s}}\right)$$
$$= 341.0\text{ m/s}$$

EVALUATE: Green's was the first supersonic land speed record (the speed of sound in air is about 340 m/s). This example shows a useful rule of thumb: A speed expressed in m/s is a bit less than half the value expressed in mi/h, and a bit less than one-third the value expressed in km/h. For example, a normal freeway speed is about 30 m/s = 67 mi/h = 108 km/h, and a typical walking speed is about 1.4 m/s = 3.1 mi/h = 5.0 km/h.

Example 1.2 **Converting volume units**

The world's largest cut diamond is the First Star of Africa (mounted in the British Royal Sceptre and kept in the Tower of London). Its volume is 1.84 cubic inches. What is its volume in cubic centimeters? In cubic meters?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Here we are to convert the units of a volume from cubic inches (in.^{3}) to both cubic centimeters (cm^3) and cubic meters (m^3). Appendix E gives us the equality 1 in. = 2.540 cm, from which we obtain 1 in.^{3} = (2.54 cm)^3. We then have

$$1.84 \text{ in.}^3 = (1.84 \text{ in.}^3)\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right)^3$$
$$= (1.84)(2.54)^3 \frac{\cancel{\text{in.}^3}\ \text{cm}^3}{\cancel{\text{in.}^3}} = 30.2 \text{ cm}^3$$

Appendix F also gives us 1 m = 100 cm, so

$$30.2 \text{ cm}^3 = (30.2 \text{ cm}^3)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3$$
$$= (30.2)\left(\frac{1}{100}\right)^3 \frac{\cancel{\text{cm}^3}\ \text{m}^3}{\cancel{\text{cm}^3}} = 30.2 \times 10^{-6} \text{ m}^3$$
$$= 3.02 \times 10^{-5} \text{ m}^3$$

EVALUATE: Following the pattern of these conversions, you can show that 1 in.$^3 \approx$ 16 cm^3 and that 1 m$^3 \approx$ 60,000 in.^{3}. These approximate unit conversions may be useful for future reference.

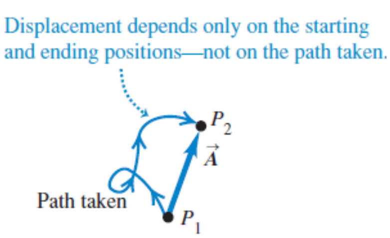
Vectors

Some physical quantities, such as time, temperature, mass, and density, can be described completely by a single number with a unit. But many other important quantities in physics have a direction associated with them and cannot be described by a single number. A simple example is describing the motion of an airplane: We must say not only how fast the plane is moving but also in what direction. The speed of the airplane combined with its direction of motion together constitutes a quantity called velocity.

When a physical quantity is described by a single number, we call it a **scalar quantity**. In contrast, a **vector quantity** has both a **magnitude** (the “how much” or “how big” part) and a **direction** in space.

For example, **Displacement** is a vector quantity, always a straight-line segment directed from the starting point to the ending point, even if the path is curved,

- We usually represent a vector quantity such as displacement by a single letter ($\vec{A}$), symbols in ***boldface italic type with an arrow above them***.
- We always draw a vector as a line with an arrowhead at its tip. The length of the line shows the vector’s magnitude, and the direction of the line shows the vector’s direction.
- If two vectors have the same direction, they are ***parallel***.
- If they have the same magnitude and the same direction, they are ***equal***, no matter where they are located in space.
- We define the negative of a vector ($-\vec{A}$) as a vector having the same magnitude as the original vector but the ***opposite direction***
- When two vectors $\vec{A}$ and $\vec{B}$ have opposite directions, whether their magnitudes are the same or not, we say that they are **antiparallel**, e.g. $\vec{A} = -\vec{B}$
- We usually represent the magnitude of a vector quantity (in the case of a displacement vector, its length) by the same letter used for the vector, but in light italic type with no arrow on top, A.
- The magnitude of a vector quantity is a scalar quantity (a number) and is always positive.
- Note that a **vector** can never be equal to a **scalar** because they are different kinds of quantities, the expression, $\vec{A}$ = 6 m” is just as wrong as 2 oranges = 3 apples!
- When drawing diagrams with vectors, it’s best to use a scale similar to those used for maps. For example, a displacement of 5 km might be represented in a diagram by a vector 1 cm long and a displacement of 10 km by a vector 2 cm long.



Reference; University Physics with Modern Physics-Sears et.al.

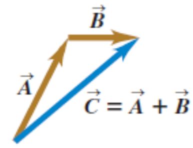
Vector Addition and Subtraction

We call displacement $\vec{C}$ the **vector sum,** or **resultant,** of displacements $\vec{A}$ and $\vec{B}$, we express this relationship symbolically as

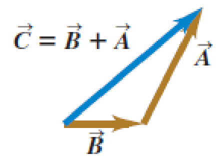
$\vec{C} = \vec{A} + \vec{B}$

Three ways to add two vectors, as shown in the figures, the order in vector addition doesn't matter; vector addition is **commutative**.

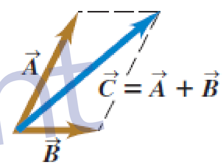
- **We can add two vectors by placing them head to tail.**



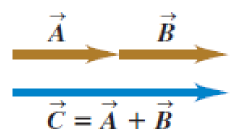
- **Adding them in reverse order gives the same result.**



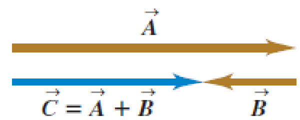
- **We can also add them by constructing a parallelogram.**



- **Only when two vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their sum equal the sum of their magnitudes:** ***C=A+B***

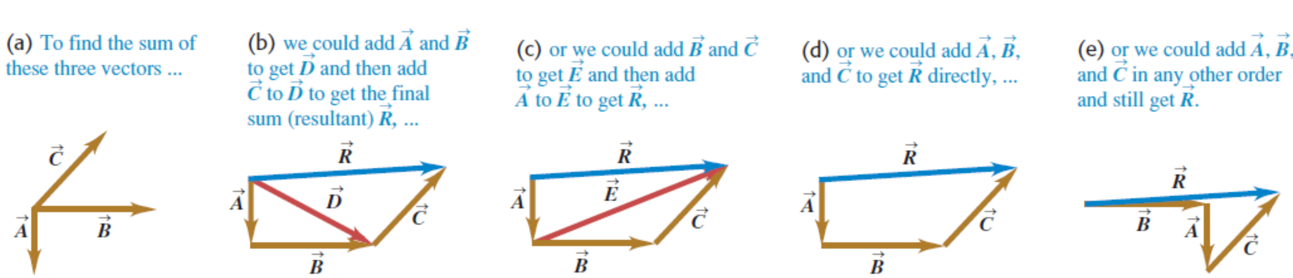


- **When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their sum equals the difference of their magnitudes**

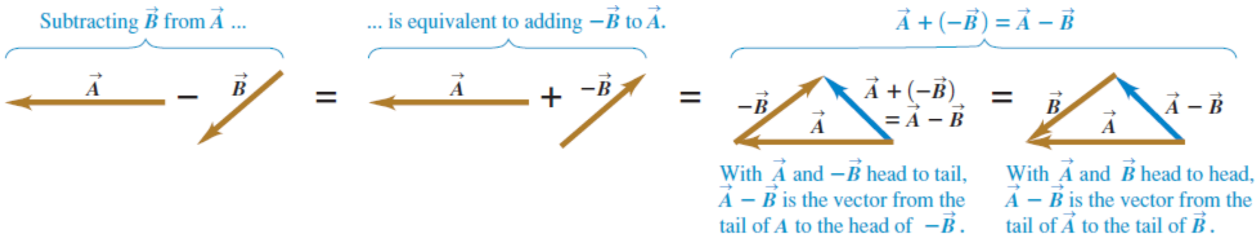


- When we need to add more than two vectors, we may first find the sum of any two vectors; then add sum to the third, and so on.

Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$,



- To construct the vector difference $\vec{A} - \vec{B}$ you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.



Example for adding two vectors at right angles;
A cross-country skier skis 1.00 km north and then 2.00 km east on a horizontal snowfield. How far and in what direction is she from the starting point?

The distance from the starting point to the ending point is equal to the length of the hypotenuse:

$$\sqrt{(\mathbf{1.00}\boldsymbol{km})^2 + (\mathbf{2.00}\boldsymbol{km})^2} = \mathbf{2.24\ km}$$

$$\boldsymbol{tan}\,\emptyset = \frac{\mathbf{2}\boldsymbol{km}}{\mathbf{1}\boldsymbol{km}}$$

$$\emptyset = \mathbf{63.4}°$$

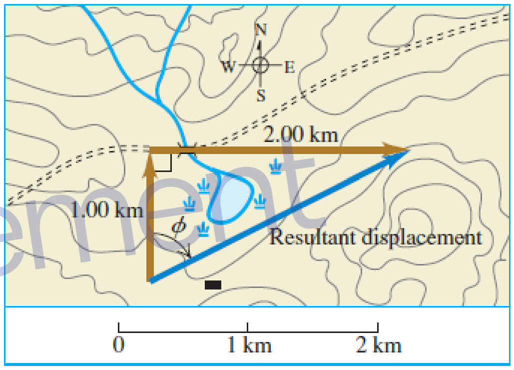
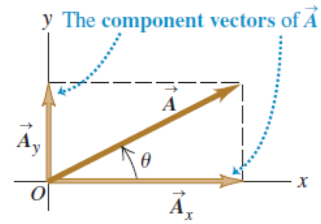


Figure 1.Scale diagram of the two displacements and the resultant net displacement

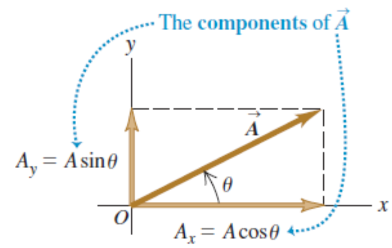
We can describe the direction as 63.4° east of north
or 90° - 63.4° = 26.6°north of east.

Components of Vectors

In previous Section, we added vectors by using a scale diagram and by using properties of right triangles. Measuring a diagram offers only very limited accuracy, and calculations with right triangles work only when the two vectors are perpendicular. So we need a simple but general method for adding vectors. This is called the method of components.

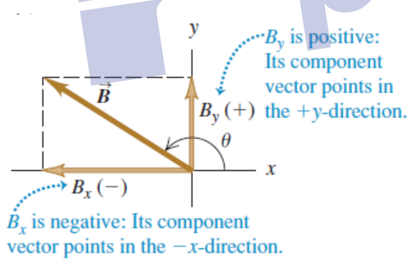


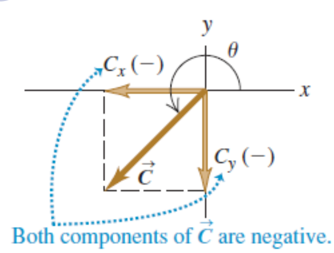
- Any set of vectors which, when added, give vector $\vec{\boldsymbol{A}}$ are called the components of $\vec{\boldsymbol{A}}$
- Representing a vector $\vec{\boldsymbol{A}}$ in terms of components; vectors $\vec{\boldsymbol{A}}_x$ and $\vec{\boldsymbol{A}}_y$



- Components A_x and A_y (which in this case are both positive).
  - $\frac{A_x}{A} = \cos\theta, \quad \gg \boxed{A_x = A\cos\theta},$
  - $\frac{A_y}{A} = \sin\theta, \quad \gg \quad \boxed{A_y = A\sin\theta}$ (1)

- If the rotation of the angle is from the (+*x-axis*) toward the (+*y-axis*) as shown in the figures, then θ is ***positive***
- If the rotation is from the (+*x-axis*) toward the (-*y*-axis), θ is ***negative***
- The components of a vector may be positive or negative numbers as shown in these figures;





- *Equations* (1) are correct only when the angle is measured from the **positive** *x-axis* as described above.

***Example*-1;**

(a) What are the ***x***- and ***y***-components of vector $\vec{\boldsymbol{D}}$ in Fig. 1 a?, $\boldsymbol{D} = 3.00$, $\alpha = 45°$
(b) What are the ***x***- and ***y***-components of vector $\vec{\boldsymbol{E}}$ in Fig. 1 b?, $\boldsymbol{E} = 4.50$ m, $\beta = 37.0°$

Solution;

a) The angle we must use in Eqs. (1) is $\theta = -\alpha = -45°$, we then find

$$\boldsymbol{D}_X = \boldsymbol{D}\cos\theta = (3.00\text{ m})\cos(-45°) = +2.1\text{ m}$$

$D_y = D \sin\theta = (3.00\text{ m}) \sin(-45°) = -2.1$ m

b) $\theta = 90.0° - \beta = 90.0° - 37.0° = 53.0°$, then

$E_x = E \cos 53.0° = (4.50\text{ m}) \cos(53.0°) = +2.71$ m
$E_y = E \sin 53.0° = (4.50\text{ m}) \sin(53.0°) = +3.59$ m

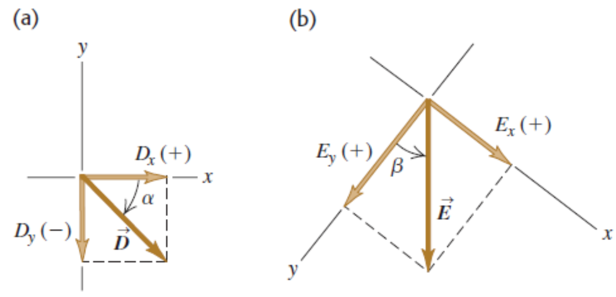


Figure 1. Calculating the x- and y-components of vectors

Vector Calculations Using Components

1- The vector ***magnitude*** and ***direction***
The ***magnitude*** of vector $\vec{A}$ is

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}} \tag{2}$$

The vector ***direction*** can be calculated from;

$\tan\theta = \frac{A_y}{A_x}$, and $\theta = \arctan\frac{A_y}{A_x}$, or $\tan^{-1}(\frac{A_y}{A_x})$ is also commonly used (3)

- **Note:** *Any two angles that differ by 180° have the same tangent.*

***Example-2*;**
$A_x = 2m$ and $A_y = -2m$ as in Fig. 2; then $\tan\theta = -1$. But both 135° and 315° or (-45°) have tangents of -1.
To decide which is correct, we have to look at the individual components. Because A_x is positive and A_y is negative, the angle must be in the fourth quadrant;
Thus $\theta = 315°$ (or -45°) is the correct value.

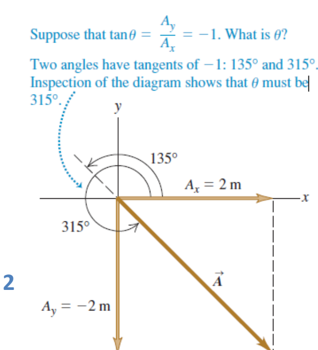


Figure 2

2- ***Multiplying*** a vector by a ***scalar***

If we multiply a vector $\vec{A}$ by a scalar c, each component of the product $\vec{D} = c\vec{A}$
$D_x = cA_x$, & $D_y = cA_y$

3- ***Calculate the vector sum*** (**resultant**) of two or more vectors

Figure 3 shows two vectors $\vec{A}$ and $\vec{B}$ and their vector sum $\vec{R}$, along with the x- and y-components of all three vectors.

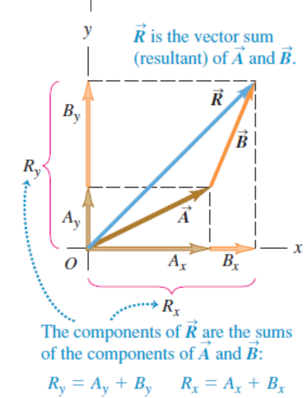


Figure 3

$R_x = A_x + B_x$ $\quad\quad\quad$ $R_y = A_y + B_y$ (components of $\vec{R} = \vec{A} + \vec{B}$)

- If we know the components of any two vectors, we can compute the components of the vector sum by using *Eqs*. (1)
- The magnitude and direction can be obtained from *Eqs*. (2) and (3)
- We can extend this procedure to find the sum of any number of vectors, for example, If $\vec{R}$ is the vector sum of $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, $\vec{E}$,… the components of $\vec{R}$ are

  $R_x = A_x + B_x + C_x + D_x + E_x + \ldots$
  $R_y = A_y + B_y + C_y + D_y + E_y + \ldots$

- These vectors lie in the *xy-plane*, but the component method works for vectors having any direction in space, so we need introducing a *z-axis* perpendicular to the *xy-plane*.
- In general a vector $\vec{A}$ has components A_x, A_y and A_z in the three coordinate directions, its magnitude is; $A = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$ (4)

Example 3, the following three displacements lead to point in a field, find the vectors sum $\vec{R}$ (resultant) and direction ($\boldsymbol{\theta}$) of the three displacements

$\vec{A}$: 72.4 m, 32.0° east of north
$\vec{B}$: 57.3 m, 36.0° south of west
$\vec{C}$: 17.8 m due south

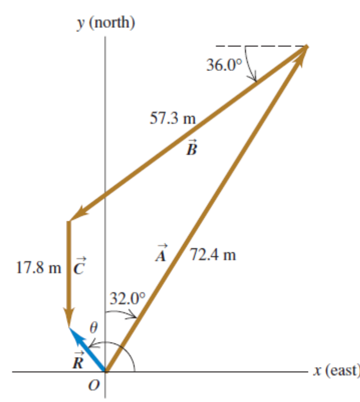
Solution:
The angles of the vectors, measured from the +*x*-axis toward the +*y*-axis, are (90.0° - 32.0°) = 58.0°, (180.0° + 36.0°) =216.0°, and 270.0°, respectively. We may now use *Eqs*. (1) to find the components of $\vec{A}$
$A_y = A \sin\theta = 72.4$ m (sin 58.0°2) = 61.40 m
$A_x = A \cos\theta = 72.4$ m (cos 58.0°) = 38.37 m

The table below shows the components of all the displacements, the addition of the components, and the other calculations.

Distance	Angle	*x*-component	*y*-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$

$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = -51°$$



The correct value is θ = 180° - 51° = 129°, or 39° west of north.

Example 4, a simple vector addition in three dimensions;
After an airplane takes off, it travels 10.4 km west, 8.7 km north, and 2.1 km up. How far is it from the take-off point?
Solution;
Let the +x-axis be east, the +y-axis north, and the +z-axis up. Then the components of the airplane's displacement are Ax = -10.4 km, Ay = 8.7 km, and Az = 2.1 km.
From Eq. (1), the magnitude of the displacement is

$$A = \sqrt{(-10.4)^2 + (8.7)^2 + (2.1)^2} = 13.7 \text{ km}$$

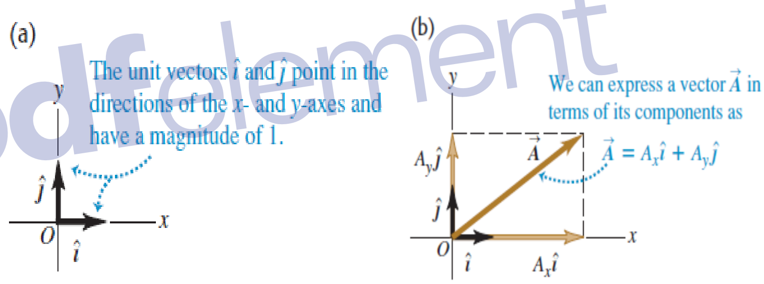
Test Your Understanding of previous Section

Two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ both lie in the *xy-plane*. (a) Is it possible for $\vec{\boldsymbol{A}}$ to have the same magnitude as $\vec{\boldsymbol{B}}$ but different components? (b) Is it possible for $\vec{\boldsymbol{A}}$ to have the same components as $\vec{\boldsymbol{B}}$ but a different magnitude?

Unit Vectors

A **unit vector** is a vector that has a magnitude of 1, with no units. Its only purpose is to *point*—that is, to describe a direction in space.

- A unit vector $\hat{\boldsymbol{\imath}}$ for the positive ***x-axis***
- A unit vector $\hat{\boldsymbol{\jmath}}$ for the positive ***y-axis***
- A unit vector $\hat{\boldsymbol{k}}$ for the positive ***z-axis***



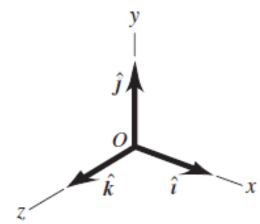
We can express the vector sum $\vec{\boldsymbol{R}}$ of two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ as follows:

$$\begin{aligned}
\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath} \\
\vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath} \\
\vec{R} &= \vec{A} + \vec{B} \\
&= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) \\
&= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} \\
&= R_x\hat{\imath} + R_y\hat{\jmath}
\end{aligned}$$

- If the vectors do not all lie in the xy-plane, then we need a third component $\hat{k}$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$
$$= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$$



***Example 1*,** given the two below displacements, fined the magnitude of the displacement $\mathbf{2\vec{D} - \vec{E}}$

$\vec{D} = (6.00\hat{\imath} + 3.00\hat{\jmath} - 1.00\hat{k})$ m and

$\vec{E} = (4.00\hat{\imath} - 5.00\hat{\jmath} + 8.00\hat{k})$ m

Solution;

From Eq.4 we find the magnitude of $\vec{F}$

$$\vec{F} = 2(6.00\hat{\imath} + 3.00\hat{\jmath} - 1.00\hat{k})\text{ m} - (4.00\hat{\imath} - 5.00\hat{\jmath} + 8.00\hat{k})\text{ m}$$
$$= [(12.00 - 4.00)\hat{\imath} + (6.00 + 5.00)\hat{\jmath} + (-2.00 - 8.00)\hat{k}]\text{ m}$$
$$= (8.00\hat{\imath} + 11.00\hat{\jmath} - 10.00\hat{k})\text{ m}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(8.00\text{ m})^2 + (11.00\text{ m})^2 + (-10.00\text{ m})^2}$$
$$= 16.9\text{ m}$$

Test Your Understanding of above Section; arrange the following vectors in order of their magnitude, with the vector of largest magnitude first. (i) $\vec{A} = (3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (ii) $\vec{B} = (-3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (iii) $\vec{C} = (3\hat{\imath} - 5\hat{\jmath} - 2\hat{k})$ m; (iv) $\vec{D} = (3\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$ m.

Products of Vectors

Vectors are not ordinary numbers, so ordinary multiplication is not directly applicable to vectors.
We will define two different kinds of products of vectors. The first, called the ***scalar product,*** yields a result that is a scalar quantity.
The second, the ***vector product,*** yields another vector.

1-Scalar Product or Dot product

- The ***scalar product*** of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}.\vec{B}$ Because of this notation, the scalar product is also called the ***dot product.***

- Although $\vec{A}$ and $\vec{B}$ are *vectors*, the quantity $\vec{A}.\vec{B}$ is a ***scalar***.

- $\vec{A}.\vec{B} = \vec{B}.\vec{A}$ *(scalar product is commutative)*

- $\vec{A}\cdot\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$ (definition of the scalar (dot) product)

  (5)

- $\vec{A}.\vec{B} = A(B\cos\emptyset)$, *(Magnitude of A) times (Component of B in direction of A)*
  - $\vec{A}.\vec{B} = B(A\cos\emptyset)$, *(Magnitude of B) times (Component of A in direction of B)*

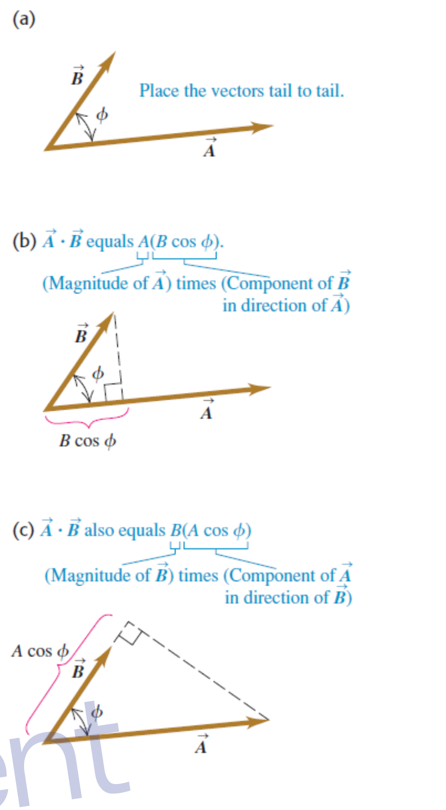


Figure 4 Calculating the scalar product of two vectors

- The scalar product can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$.

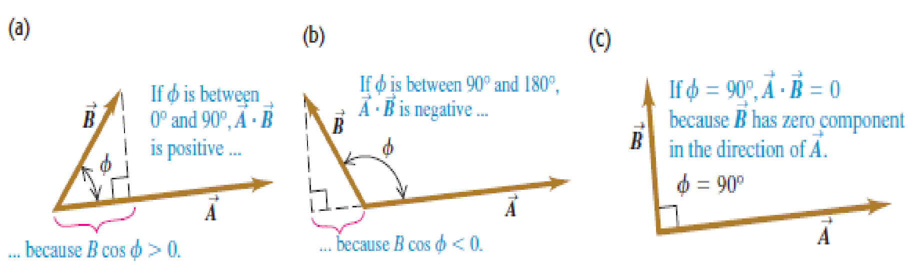


Figure 5 The scalar product can be positive, negative, or zero

1.1 Scalar Product Using Components

Now we express $\vec{A}$ and $\vec{B}$ in terms of their components, expand the product, and use these products of unit vectors:

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{(scalar (dot) product in terms of components)} \tag{6}$$

Thus *the scalar product of two vectors is the sum of the products of their respective components.*

- The scalar product gives a straightforward way to find the angle $\emptyset$ between any two vectors $\vec{A}$ and $\vec{B}$ whose components are known, as following;

$$\boxed{\vec{A}\,.\vec{B} = |A||B|(\cos\emptyset) \qquad \gg \cos\emptyset = \frac{\vec{A}\,.\vec{B}}{|A||B|}}$$

Example 1; Find the scalar product of the two vectors $\vec{A}\,.\vec{B}$ in Fig. 6.
The magnitudes of the vectors are $A = 4.00$ and $B = 5.00$.
Solution;
We can calculate the scalar product in two ways:
1) Using the magnitudes of the vectors and the angle between them (*Eq*. 5),
2) Using the components of the vectors (*Eq*. 6),
Do it in both ways!

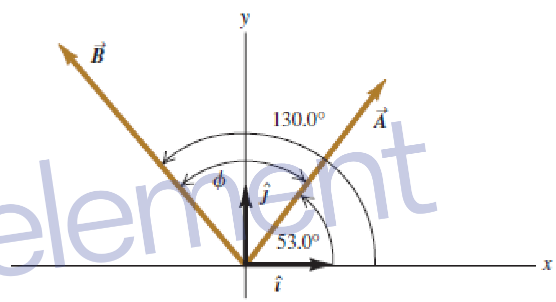


Figure 6

Example 2; find θ between $\vec{A}$ *and* $\vec{B}$?

$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\quad \vec{B} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

Example 3; if $\theta = 30^o$ between $\vec{A}$ *and* $\vec{B}$ find q value**?**

$\vec{A} = 2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$, $\quad \vec{B} = q\hat{\imath} + \hat{\jmath} - 2\hat{k}$

Example 4; Find the angle between the vectors

$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + 1\hat{k}$, $\quad \vec{B} = -4\hat{\imath} + 2\hat{\jmath} - 1\hat{k}$

2- Vector Product or cross product

- The ***vector product*** of two vectors $\vec{A}$ ***and*** $\vec{B}$ also called the ***cross product***,
- It is denoted by $\vec{A} \times \vec{B}$,
- The result of two vectors ($\vec{A} \times \vec{B}$) is a ***vector*** ($\vec{C}$), **and its** perpendicular to $\vec{A}$ & $\vec{B}$
- $\vec{A} \times \vec{B} = \vec{C}$, the magnitude of ***the vector product (C)*** **is** $\boxed{C = AB\sin\emptyset}$ (7)
- The vector product determined by the right-hand rule.
- The vector product is anti-commutative.

$$\vec{A} \times \vec{B} = -\vec{A} \times \vec{B}$$

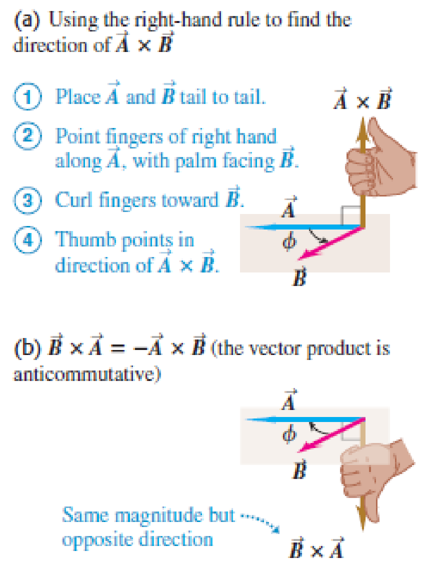


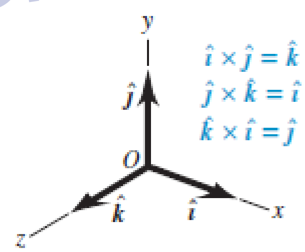
Figure 7 The vector product is determined by the right-hand rule

2.1 *Vector Product Using Components*

- $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$
- $\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$

  $\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$

  $\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$



- $\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$ (8)
- Thus the components of $\vec{C} = \vec{A} \times \vec{B}$ are given by

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x$$

$$(\text{components of } \vec{C} = \vec{A} \times \vec{B})$$

- The vector product can also be expressed in ***determinant*** form as
- $\vec{A} \times \vec{B} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$

Example,

Vector $\vec{A}$ has magnitude 6 units and is in the direction of the $+x$-*axis*. Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$-*axis* (Fig. 8). Find the vector product $\vec{C} = \vec{A} \times \vec{B}$.

Solution;

We'll find the vector product in two ways, which will provide a check of our calculations. First we'll use Eq. (7) and the right-hand rule; then we'll use Eq. (8) to find the vector product using components.

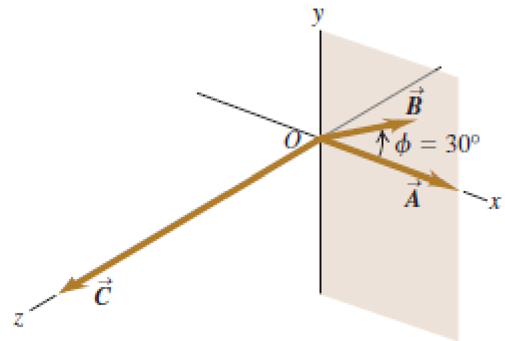


Figure 8

From Eq. (7); $C = AB\, sin\emptyset,$ the magnitude of the vector product is

$$(\mathbf{6})(\mathbf{4})(\boldsymbol{sin30}) = \mathbf{12}$$

By the right-hand rule, the direction of $\vec{A} \times \vec{B}$ is along the +z-axis (the direction of the unit vector $\hat{k}$), so we have $\vec{C} = \vec{A} \times \vec{B}$=$12\hat{k}$

To use Eq. (8), we first determine the components of $\vec{A}$ *and* $\vec{B}$

$A_x = 6$ $\quad A_y = 0$ $\quad A_z = 0$

$B_x = 4\, cos30 = 2\sqrt{3}$ $\quad B_y = 4\, sin30 = 2$ $\quad B_z = 0$

Then eq. (8) yield

$\boldsymbol{C_x} = (0)(0) - (0)(2) = 0$

$\boldsymbol{C_y} = (0)(2\sqrt{3}) - (6)(0) = 0$

$\boldsymbol{C_z} = (6)(2) - (0)(2\sqrt{3}) = 12$

Thus again we have $\vec{C} = 12\hat{k}$

Test Your Understanding of the Section;

Vector $\vec{A}$ has magnitude 2 and vector $\vec{B}$ has magnitude 3. The angle $\emptyset$ between $\vec{A}$ and $\vec{B}$ is known to be 0°, 90°, or 180°.

For each of the following situations, state what the value of $\emptyset$ must be. (In each situation there may be more than one correct answer.

(a) $\vec{A}.\vec{B} = \mathbf{0}$, (b) $\vec{A} \times \vec{B} = \mathbf{0}$, (c) $\vec{A}.\vec{B} = \mathbf{6}$, **(d)** $\vec{A}.\vec{B} = \mathbf{-6}$, (e) (Magnitude of $\vec{A} \times \vec{B} = 6$)